where α is defined as the mass fraction of the second phase and v_i is specific volume of the i-th phase at p and T.

Conditions 4 and 5 imply that the total internal energy of a mass element is the sum of the internal energies of the two phases:

$$E = (1-\alpha)E_1 + \alpha E_2 \tag{3.7}$$

where E_i is specific internal energy of the i-th phase at p and T.

In order to obtain a suitable constitutive relation when the phase transition has a finite reaction rate, we assume first that the p,v,E surfaces in each phase can be extended smoothly into metastable regions overlapping the equilibrium region of mixed phase, as in Fig. 3.1.

Then we have the following functional forms:

$$\begin{aligned} v_1 &= v_1(p,T) \\ v_2 &= v_2(p,T) \\ E_1 &= E_1(p,T) \\ E_2 &= E_2(p,T). \end{aligned} \tag{3.8}$$

Then from Eqs. (3.6) and (3.7) we have

$$dv = (1-\alpha)dv_1 + \alpha dv_2 + (v_2-v_1)d\alpha \tag{3.9}$$

$$dE = (1-\alpha)dE_1 + \alpha dE_2 + (E_2-E_1)d\alpha. \tag{3.10}$$

From Eq. (3.8):

$$dv_i = (\partial v_i/\partial T)_p dT + (\partial v_i/\partial p)_T dp \tag{3.11}$$

$$i = 1 \text{ or } 2.$$

With E a function of T and p we have:

$$dE_i = (\partial E_i/\partial T)_p dT + (\partial E_i/\partial p)_T dp$$

$$= (C_{pi} - p(\partial v_i/\partial T)_p)dT - (T(\partial v_i/\partial T)_p + p(\partial v_i/\partial p)_T)dp \tag{3.12}$$

$$i = 1 \text{ or } 2$$

where C_{pi} = specific heat of i-th phase at constant pressure.

Substituting (3.11) and (3.12) into (3.9) and (3.10), we get

$$dv = \ell_1 dp + m_1 dT + n_1 d\alpha \tag{3.13}$$

$$dE = \ell_2 dp + m_2 dT + n_2 d\alpha \tag{3.14}$$

where

$$\ell_1 = (1-\alpha)\ (\partial v_1/\partial p)_T + \alpha(\partial v_2/\partial p)_T \tag{3.15}$$

$$m_1 = (1-\alpha)\ (\partial v_1/\partial T)_p + \alpha(\partial v_2/\partial T)_p \tag{3.16}$$

$$n_1 = v_2 - v_1 \tag{3.17}$$

$$\ell_2 = -(1-\alpha)\ (T(\partial v_1/\partial T)_p + p(\partial v_1/\partial p)_T) - \alpha\ (T(\partial v_2/\partial T)_p + p(\partial v_2/\partial p)_T) \tag{3.18}$$

$$m_2 = (1-\alpha)\ (C_{p1} - p(\partial v_1/\partial T)_p) + \alpha(C_{p2} - p(\partial v_2/\partial T)_p) \tag{3.19}$$

$$n_2 = E_2 - E_1 \tag{3.20}$$

and

$$dE = -(p+q)dv \text{ from Eq. (3.5).} \tag{3.21}$$

Therefore, in principle, (3.13) and (3.14) can be solved for dp and dT if $d\alpha$ is given.

Now we assume the following relaxation